Hoofdstuk 7: Exponenten en logaritmen

7.1 Machten met gehele exponenten

Opgave 1:

a. ja
b. nee
c. ja

Opgave 2:

a. $2a^2 \cdot 4a^3 = 8a^5$
b. $-5a^7 \cdot a^3 = -5a^{10}$
c. $\frac{-28a^6}{7a} = -4a^5$
d. $(-4a)^4 = 256a^4$
e. $-(3a^4)^2 = -9a^8$
f. $(-2a^2)^5 = -32a^{10}$
g. $(-a^3)^3 = -a^9$
h. $(5a)^3 \cdot -3a = 125a^3 \cdot -3a = -375a^4$
i. $\left(\frac{9a^4}{a}\right)^2 = (9a^3)^2 = 81a^6$

Opgave 3:

a. $(ab)^4 \cdot a = a^4b^4 \cdot a = a^5b^4$
b. $(-2ab)^3 \cdot b = -8a^3b^3 \cdot b = -8a^3b^4$
c. $(3a)^2 + (2b)^2 = 9a^2 + 4b^2$
d. $(3a)^3 - 8a^3 = 27a^3 - 8a^3 = 19a^3$
e. $(\frac{1}{2}a)^2 + (-a)^2 = \frac{1}{4}a^2 + a^2 = 1\frac{1}{4}a^2$
f. $(5a^4)^2 + (-a^2)^4 = 25a^8 + a^8 = 26a^8$

Opgave 4:

a. de grafieken zijn hetzelfde
b. $y = \frac{1}{x}$
c. $x^0 = 1$

Opgave 5:

a. de exponenten zijn steeds 1 hoger (of lager)
   de uitkomst wordt steeds verdubbeld (of gehalveerd)
b. $2^1 = 2$
   $2^0 = 2 : 2 = 1$
   $2^{-1} = 1 : 2 = \frac{1}{2}$
   $2^{-2} = \frac{1}{2} : 2 = \frac{1}{4}$

c. $2^{-3} = \frac{1}{8}$

$2^{-4} = \frac{1}{16}$

Opgave 6:

a. $a^{-5} \cdot a^{2} = a^{-3}$

b. $a^{4} \cdot \frac{1}{a^{6}} = a^{4} \cdot a^{-6} = a^{-2}$

c. $\frac{a^{3}}{a^{-2}} = a^{5}$

d. $(a^{-3})^{4} = a^{-12}$

e. $a^{4} : \frac{1}{a^{3}} = a^{4} \cdot \frac{a^{3}}{1} = a^{7}$

f. $a^{7} : a^{0} = a^{7}$

g. $\left(\frac{1}{a^{2}}\right)^{3} = (a^{-2})^{3} = a^{-6}$

h. $1 = a^{0}$

i. $a^{3} \cdot (a^{4})^{-2} = a^{3} \cdot a^{-8} = a^{-5}$

Opgave 7:

a. $4^{-3} = \frac{1}{4^{3}} = \frac{1}{64}$

b. $\left(\frac{5}{6}\right)^{-2} = \frac{5^{-2}}{6^{-2}} = \frac{6^{2}}{5^{2}} = \frac{36}{25}$

c. $(3^{-1})^{4} = 3^{-4} = \frac{1}{3^{4}} = \frac{1}{81}$

d. $\left(\frac{2}{5}\right)^{-2} = \frac{2^{-2}}{5^{-2}} = \frac{5^{2}}{2^{2}} = \frac{25}{4}$

e. $(2\frac{1}{2})^{-2} = \left(\frac{5}{2}\right)^{-2} = \frac{5^{-2}}{2^{-2}} = \frac{2^{2}}{5^{2}} = \frac{4}{25}$

f. $1 : \left(\frac{3}{7}\right)^{-2} = 1 : \frac{3^{-2}}{7^{-2}} = 1 : \frac{7^{2}}{3^{2}} = 1 : \frac{49}{9} = \frac{9}{49}$

Opgave 8:

a. $6a^{-3} \cdot b^{2} = \frac{6b^{2}}{a^{3}}$

b. $\frac{1}{3}a^{-4} = \frac{1}{3a^{4}}$

c. $2a^{-3} = \frac{2}{a^{3}}$

d. $3a \cdot b^{-2} = \frac{3a}{b^{2}}$

e. $3a^{-2} \cdot b^{3} = \frac{3b^{3}}{a^{2}}$

f. $(3a)^{-2} \cdot 2b^{-1} = 3^{-2} \cdot a^{-2} \cdot 2 \cdot b^{-1} = \frac{2}{9a^2b}$

7.2 Machten met gebroken exponenten.

Opgave 9:

a. ja

b. de grafieken zijn gelijk

Opgave 10:

a. $5^{\frac{1}{3}} = \sqrt[3]{5}$

b. $7^{-\frac{1}{3}} = \frac{1}{7^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{7}}$

c. $2a^{\frac{2}{5}} = 2 \cdot \sqrt[5]{a^2}$

d. $-a^{-\frac{3}{5}} = \frac{-1}{a^{\frac{3}{5}}} = \frac{-1}{\sqrt[5]{a^3}}$

e. $4a^{-2}b^{\frac{1}{2}} = \frac{4b^{\frac{1}{2}}}{a^2} = \frac{4\sqrt{b}}{a^2}$

f. $3a^{\frac{1}{3}}b^{-\frac{1}{2}} = \frac{3a^{\frac{1}{3}}}{b^{\frac{1}{2}}} = \frac{3 \cdot \sqrt[3]{a}}{\sqrt{b}}$

Opgave 11:

a. $\sqrt{x} = x^{\frac{1}{2}}$

b. $x \cdot \sqrt[4]{x} = x^1 \cdot x^{\frac{1}{4}} = x^{1\frac{1}{4}}$

c. $\frac{1}{\sqrt[3]{x^2}} = \frac{1}{x^{\frac{2}{3}}} = x^{-\frac{2}{3}}$

d. $\sqrt[5]{\frac{1}{x}} = \sqrt[5]{x^{-1}} = x^{-\frac{1}{5}}$

e. $\frac{\sqrt{x}}{\sqrt[3]{x}} = \frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}} = x^{\frac{1}{6}}$

f. $\sqrt{x} \cdot \sqrt[3]{x^2} = x^{\frac{1}{2}} \cdot x^{\frac{2}{3}} = x^{1\frac{1}{6}}$

g. $\frac{x^2}{\sqrt[4]{x^3}} = \frac{x^2}{x^{\frac{3}{4}}} = x^{1\frac{1}{4}}$

h. $\frac{x^2 \cdot \sqrt[3]{x}}{\sqrt{x}} = \frac{x^2 \cdot x^{\frac{1}{3}}}{x^{\frac{1}{2}}} = \frac{x^{2\frac{1}{3}}}{x^{\frac{1}{2}}} = x^{1\frac{5}{6}}$

Opgave 12:

a. $8\sqrt{2} = 2^3 \cdot 2^{\frac{1}{2}} = 2^{3\frac{1}{2}}$

b. $\frac{8}{\sqrt{2}} = \frac{2^3}{2^{\frac{1}{2}}} = 2^{2\frac{1}{2}}$

c. $\frac{1}{2} \cdot \sqrt[3]{2} = 2^{-1} \cdot 2^{\frac{1}{3}} = 2^{-\frac{2}{3}}$

d. $\frac{4\sqrt{2}}{\sqrt[3]{2}} = \frac{2^2 \cdot 2^{\frac{1}{2}}}{2^{\frac{1}{3}}} = \frac{2^{2\frac{1}{2}}}{2^{\frac{1}{3}}} = 2^{2\frac{1}{6}}$

e. $\sqrt[4]{\frac{1}{9}} = \sqrt[4]{\frac{1}{3^2}} = \sqrt[4]{3^{-2}} = 3^{-\frac{2}{4}} = 3^{-\frac{1}{2}}$

f. $\frac{1}{100}\sqrt{10} = \frac{1}{10^2} \cdot 10^{\frac{1}{2}} = 10^{-2} \cdot 10^{\frac{1}{2}} = 10^{-1\frac{1}{2}}$

g. $\frac{1}{8} \cdot \sqrt[3]{\frac{1}{4}} = \frac{1}{2^3} \cdot \sqrt[3]{\frac{1}{2^2}} = 2^{-3} \cdot \sqrt[3]{2^{-2}} = 2^{-3} \cdot 2^{-\frac{2}{3}} = 2^{-3\frac{2}{3}}$

h. $10 \cdot \sqrt[3]{0{,}1} = 10^1 \cdot \sqrt[3]{10^{-1}} = 10^1 \cdot 10^{-\frac{1}{3}} = 10^{\frac{2}{3}}$

Opgave 13:

a. $x^{2\frac{1}{3}} = x^2 \cdot x^{\frac{1}{3}} = x^2 \cdot \sqrt[3]{x}$

b. $2^{2\frac{1}{2}} = 2^2 \cdot 2^{\frac{1}{2}} = 4\sqrt{2}$

c. $2^{x+3} = 2^3 \cdot 2^x = 8 \cdot 2^x$

d. $3^{x-2} = 3^{-2} \cdot 3^x = \frac{1}{3^2} \cdot 3^x = \frac{1}{9} \cdot 3^x$

Opgave 14:

a. $1{,}18^{a+5} = 1{,}18^5 \cdot 1{,}18^a = 2{,}29 \cdot 1{,}18^a$

b. $1{,}31^{a-2} = 1{,}31^{-2} \cdot 1{,}31^a = 0{,}58 \cdot 1{,}31^a$

c. $0{,}78^{a+0{,}6} = 0{,}78^{0{,}6} \cdot 0{,}78^a = 0{,}86 \cdot 0{,}78^a$

d. $1{,}15^{2a+1} = 1{,}15^1 \cdot 1{,}15^{2a} = 1{,}15 \cdot (1{,}15^2)^a = 1{,}15 \cdot 1{,}32^a$

e. $1{,}22^{2a-1} = 1{,}22^{-1} \cdot 1{,}22^{2a} = 0{,}82 \cdot (1{,}22^2)^a = 0{,}82 \cdot 1{,}49^a$

f. $8{,}35^{\frac{1}{3}a+0{,}4} = 8{,}35^{0{,}4} \cdot 8{,}35^{\frac{1}{3}a} = 2{,}34 \cdot (8{,}35^{\frac{1}{3}})^a = 2{,}34 \cdot 2{,}03^a$

g. $8{,}35^{\frac{1}{3}a} = (8{,}35^{\frac{1}{3}})^a = 2{,}03^a$

h. $0{,}72^{2(a-1{,}2)} = 0{,}72^{2a-2{,}4} = 0{,}72^{-2{,}4} \cdot 0{,}72^{2a} = 2{,}20 \cdot (0{,}72^2)^a = 2{,}20 \cdot 0{,}52^a$

Opgave 15:

a. $x^{1{,}8} = 50$
$x = \sqrt[1{,}8]{50} = 8{,}79$

b. $x^{-3} = 5$
$x = \sqrt[-3]{5} = 0{,}58$

c. $3 \cdot x^{2{,}25} + 1 = 27$
$3 \cdot x^{2{,}25} = 26$
$x^{2{,}25} = 8\frac{2}{3}$
$x = \sqrt[2{,}25]{8\frac{2}{3}} = 2{,}61$

d. $5 \cdot x^{-1} = 7$
$x^{-1} = 1{,}4$
$x = \sqrt[-1]{1{,}4} = 0{,}71$

e. $4 \cdot x^{-1{,}8} + 16 = 500$
$4 \cdot x^{-1{,}8} = 484$
$x^{-1{,}8} = 121$

$x = \sqrt[-1,8]{121} = 0{,}070$

f. $x^9 = \sqrt{3}$

$x = \sqrt[9]{\sqrt{3}} = 1{,}063$

Opgave 16:

a. $5x^{-1,2} + 7 = 19$

$5x^{-1,2} = 12$

$x^{-1,2} = 2{,}4$

$x = \sqrt[-1,2]{2{,}4} = 0{,}482$

b. $4x^{0,4} - 5 = 109$

$4x^{0,4} = 114$

$x^{0,4} = 28{,}5$

$x = \sqrt[0,4]{28{,}5} = 4336{,}228$

c. $x^{1\frac{1}{3}} = 10$

$x = \sqrt[1\frac{1}{3}]{10} = 5{,}623$

d. $\sqrt[3]{x^2} = 26$

$x^{\frac{2}{3}} = 26$

$x = \sqrt[\frac{2}{3}]{26} = 132{,}575$

e. $5 \cdot \sqrt[3]{x} = 8$

$\sqrt[3]{x} = 1{,}6$

$x = 1{,}6^3 = 4{,}096$

f. $3 \cdot \sqrt[4]{x^3} - 1 = 36$

$3 \cdot \sqrt[4]{x^3} = 37$

$\sqrt[4]{x^3} = 12\frac{1}{3}$

$x^3 = (12\frac{1}{3})^4 = 23137{,}79$

$x = \sqrt[3]{23137{,}79} = 28{,}495$

Opgave 17:

a. $B = 20 + 0{,}7 \cdot 17^{1,52} = 72$ euro

b. $20 + 0{,}7v^{1,52} = 97$

$y_1 = 20 + 0{,}7x^{1,52}$ en $y_2 = 97$ intersect geeft: $x = 22$

dus $v = 30 + 22 = 52$ $^{km}/_{uur}$

c. $y_2 = 1648$ intersect geeft $x = 164$ dus 164 $^{km}/_{uur}$

d. nee

als $v = 20$ dan $B = 86$

als $v = 40$ dan $B = 211$

Opgave 18:

a. $F = (2000 - 16{,}3 \cdot 60)(-5 - -20)^{-1,668} = 11\text{min}$

b. $(2000-16{,}3v)(-5--18)^{-1{,}668}=20$
$(2000-16{,}3)\cdot 0{,}0139=20$
$2000-16{,}3v=1442$
$-16{,}3v=-558$
$v=34\,{}^{km}\!/_{uur}$

c. $F=\frac{10}{40}=0{,}25$ uur =15min en $v=40$ (rijwind)
$(2000-16{,}3\cdot 40)(-5-T)^{-1{,}668}=15$
$y_1=1348(-5-x)^{-1{,}668}$ en $y_2=15$ intersect geeft $x=-19{,}8$
dus bij $-20°$ C of kouder

Opgave 19:

a. $P=a\cdot Q^{2{,}5}$
$8{,}1=a\cdot 3{,}2^{2{,}5}$
$8{,}1=a\cdot 18{,}3$
$a=0{,}44$

b. $y=a\cdot\dfrac{1}{x^{1{,}81}}$
$16=a\cdot\dfrac{1}{12^{1{,}81}}$
$a=16\cdot 12^{1{,}81}=1437$

Opgave 20:

a. $T=a\cdot R^{1{,}5}$
$1{,}9=a\cdot 2{,}95^{1{,}5}$
$1{,}9=a\cdot 5{,}07$
$a=0{,}37$

b. $T=0{,}37\cdot 35{,}6^{1{,}5}=78{,}6$ dagen

c. $0{,}37R^{1{,}5}=\frac{15}{24}=0{,}625$
$R^{1{,}5}=1{,}69$
$R=\sqrt[1{,}5]{1{,}69}=1{,}42$
dus $1{,}42\cdot 10^5$ km

Opgave 21:

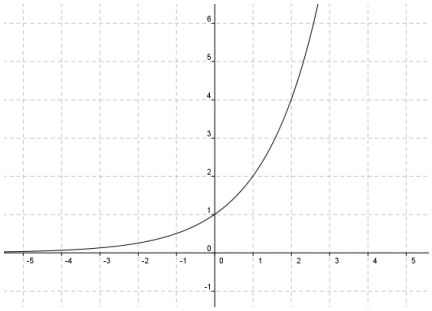
a. $W=a\cdot m^{0{,}75}$
$6700=a\cdot 40^{0{,}75}$
$6700=15{,}9a$
$a=421$

b. $W=421\cdot 4^{0{,}75}=1191$ kJ

c. $421\cdot m^{0{,}75}=50000$
$m^{0{,}75}=118{,}8$
$m=\sqrt[0{,}75]{118{,}8}=585$ kg

7.3 Exponentiële functies

Opgave 22:

a. 

b. stijgend

c. $f(-10) = 9{,}8 \cdot 10^{-4}$

$f(-20) = 9{,}5 \cdot 10^{-7}$

$f(-100) = 7{,}9 \cdot 10^{-31}$

d. als je van $x = -10$ naar $x = -500$ gaat, halveer je bij iedere stap de uitkomst, dus je uitkomst blijft positief. De Gr geeft 0.

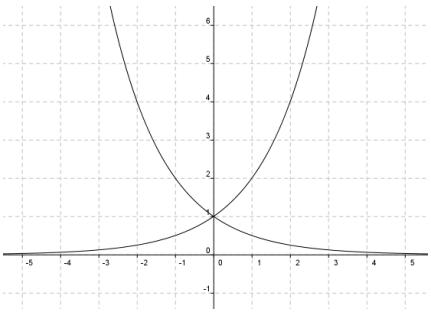
e. nee

Opgave 23:

a. dalend

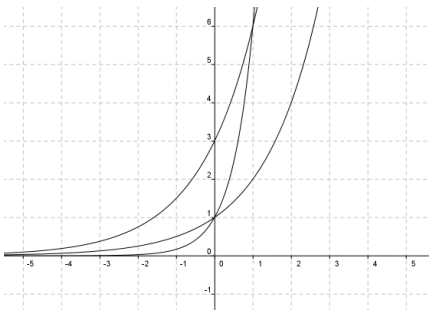
b. één oplossing , geen oplossingen

Opgave 24:

a. 

b. spiegelen in de *y*-as

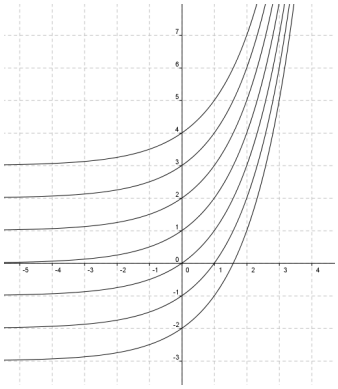
Opgave 25:

a. 

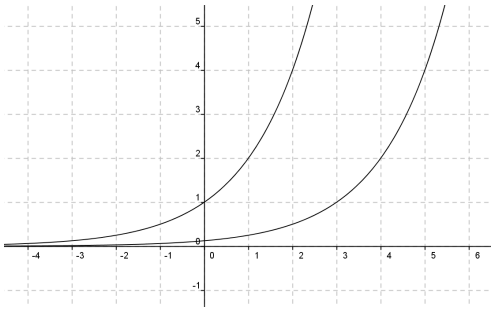
b. y_2 , de factor is 3

Opgave 26:

a.



b. translatie over $(0,5)$



Opgave 27:

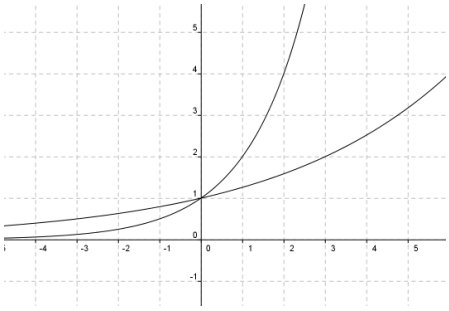
a. translatie over $(3,0)$

b. translatie over $(-4,0)$

c. translatie over $(b,0)$

Opgave 28:

a.



b. factor 3

c. factor $\frac{1}{4}$

d. er moet in het boek staan $y_4 = 2^{ax}$ dan is de factor $\frac{1}{a}$

Opgave 29:

a. $y = 3^x \xrightarrow{T(-2,-1)} y = 3^{x+2} - 1$

H.A.: $y = -1$

b. $y = 3^x \xrightarrow{T(1,5)} y = 3^{x-1} + 5$

H.A.: $y = 5$

c. $y = 0{,}5^x \xrightarrow{Vx-as,2} y = 2 \cdot 0{,}5^x \xrightarrow{T0,3)} y = 2 \cdot 0{,}5^x + 3$

H.A.: $y = 3$

d. $y = 0{,}7^x \xrightarrow{Vx-as,-3} y = -3 \cdot 0{,}7^x \xrightarrow{T(0,5)} y = -3 \cdot 0{,}7^x + 5$

H.A.: $y = 5$

e. $y = 2^x \xrightarrow{Vy-as,\frac{1}{3}} y = 2^{3x} \xrightarrow{Vx-as,3} y = 3 \cdot 2^{3x} \xrightarrow{T(0,4)} y = 3 \cdot 2^{3x} + 4$

H.A.: $y = 4$

f. $y = 0{,}8^x \xrightarrow{Vy-as,2\frac{1}{2}} y = 0{,}8^{0,4x} \xrightarrow{T(0,-10)} y = 0{,}8^{0,4x} - 10$

H.A.: $y = -10$

Opgave 30:

a. $B =< -6, \rightarrow >$ H.A.: $N = -6$

b. $B =< \leftarrow, 5 >$ H.A.: $N = 5$

c. $B =< \leftarrow, 1000 >$ H.A.: $N = 1000$

d. $B =< \leftarrow, 1000 >$ H.A.: $N = 1000$

Opgave 31:

a. $y = 3^x \xrightarrow{Vx-as,\frac{1}{2}} y = \frac{1}{2} \cdot 3^x \xrightarrow{T(0,3)} y = \frac{1}{2} \cdot 3^x + 3$

b. $y = 3^x \xrightarrow{S\ x-as} y = 3^{-x} \xrightarrow{T(0,-1)} y = 3^{-x} - 1$

c. $y = 3^x \xrightarrow{T(4,-5)} y = 3^{x-4} - 5 \xrightarrow{Vx-as,3} y = 3 \cdot 3^{x-4} - 15$

d. $y = 3^x \xrightarrow{Vx-as,3} y = 3 \cdot 3^x \xrightarrow{T(4,-5)} y = 3 \cdot 3^{x-4} - 5$

Opgave 32:

Opgave 33:

Opgave 34:

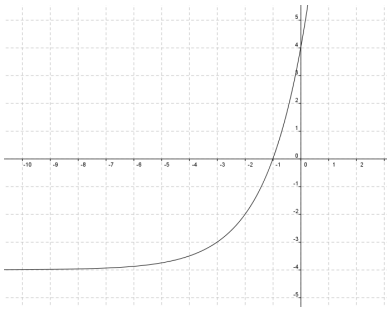
Opgave 35:

a. $y = 2^x \xrightarrow{T(-3,-4)} y = 2^{x+3} - 4$

b. $B =< -4, \rightarrow >$

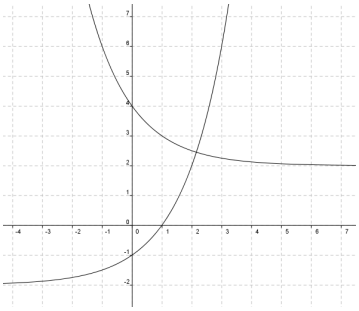
c. $y_1 = 2^{x+3} - 4$ en $y_2 = -1$
intersect geeft $x = -1{,}42$
dus $x \leq -1{,}42$

d. $f(3) = 60$
$-4 < f(x) \leq 60$



Opgave 36:

a.



$y = 2^x \xrightarrow{T(0,-2)} y = 2^x - 2$

$y = \left(\frac{1}{2}\right)^x \xrightarrow{T(1,2)} y = \left(\frac{1}{2}\right)^{x-1} + 2$

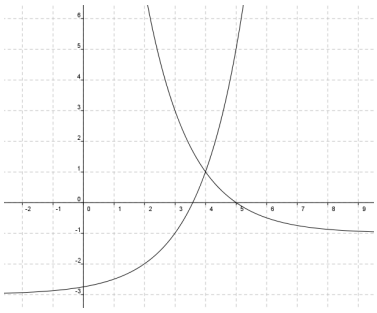
b. $B_f =< -2, \rightarrow >$

$B_g =< 2, \rightarrow >$

c. $y_1 = 2^x - 2$ en $y_2 = \left(\frac{1}{2}\right)^{x-1} + 2$ intersect geeft $x = 2{,}15$
dus $x \geq 2{,}15$

d. $p \leq -2$

Opgave 37:

a. 

b. $-3 < p \leq -1$

c. $f(2) = -2$
dus $-3 < f(x) \leq -2$

d. $f(1) = -2{,}5$
$g(1) = 63$
$AB = 63 - -2{,}5 = 65{,}5$

e. $y_1 = 2^{x-2} - 3$ en $y_2 = 4 \cdot 0{,}5^{x-3} - 1$ intersect geeft $x_P = 5$ en $x_Q = 2{,}415$
$PQ = 5 - 2{,}415 = 2{,}585$

Opgave 38:

a. $x_A = 3{,}5$

b. $2^{x-3} = \sqrt{2}$
$2^{x-3} = 2^{\frac{1}{2}}$
$x - 3 = \frac{1}{2}$
$x = 3\frac{1}{2}$

Opgave 39:

a. $2^{x+1} = 64$
$2^{x+1} = 2^6$
$x + 1 = 6$
$x = 5$

b. $2^{x-3} = \frac{1}{8}$
$2^{x-3} = \frac{1}{2^3}$
$2^{x-3} = 2^{-3}$
$x - 3 = -3$
$x = 0$

c. $2^{2x} = 2$
$2^{2x} = 2^1$
$2x = 1$
$x = \frac{1}{2}$

d. $2^x = 1$
$2^x = 2^0$
$x = 0$

e. $2^x = \frac{1}{4}\sqrt{2}$

$2^x = \frac{1}{2^2} \cdot 2^{\frac{1}{2}}$

$2^x = 2^{-1\frac{1}{2}}$

$x = -1\frac{1}{2}$

f. $2^{x+5} = 16\sqrt{2}$

$2^{x+5} = 2^4 \cdot 2^{\frac{1}{2}}$

$2^{x+5} = 2^{4\frac{1}{2}}$

$x + 5 = 4\frac{1}{2}$

$x = -\frac{1}{2}$

g. $5^{x+6} = \left(\frac{1}{5}\right)^x$

$5^{x+6} = \left(5^{-1}\right)^x$

$5^{x+6} = 5^{-x}$

$x + 6 = -x$

$2x = -6$

$x = -3$

h. $3^{2x+1} = 27\sqrt{3}$

$3^{2x+1} = 3^3 \cdot 3^{\frac{1}{2}}$

$3^{2x+1} = 3^{3\frac{1}{2}}$

$2x + 1 = 3\frac{1}{2}$

$2x = 2\frac{1}{2}$

$x = 1\frac{1}{4}$

i. $10^{2x+1} = 0{,}01$

$10^{2x+1} = 10^{-2}$

$2x + 1 = -2$

$2x = -3$

$x = -1\frac{1}{2}$

Opgave 40:

a. $2^x + 1 = 17$

$2^x = 16$

$2^x = 2^4$

$x = 4$

b. $3^x - 2 = 25$

$3^x = 27$

$3^x = 3^3$

$x = 3$

c. $5 \cdot 2^x = 80$

$2^x = 16$

$2^x = 2^4$

$x = 4$

d. $10 \cdot 3^x = 270$

$3^x = 27$

$3^x = 3^3$

$x = 3$

e. $3 \cdot 8^{2-x} = 48$

$8^{2-x} = 16$

$\left(2^3\right)^{2-x} = 2^4$

$2^{6-3x} = 2^4$

$6 - 3x = 4$

$-3x = -2$

$x = \frac{2}{3}$

f. $3 \cdot 2^x + 4 = 28$

$3 \cdot 2^x = 24$

$2^x = 8$

$2^x = 2^3$

$x = 3$

g. $5^{2x-6} = 0{,}04$

$5^{2x-6} = \frac{1}{25}$

$5^{2x-6} = \frac{1}{5^2}$

$5^{2x-6} = 5^{-2}$

$2x - 6 = -2$

$2x = 4$

$x = 2$

h. $3 \cdot 7^{3x+1} = 147$

$7^{3x+1} = 49$

$7^{3x+1} = 7^2$

$3x + 1 = 2$

$3x = 1$

$x = \frac{1}{3}$

i. $32^{x-2} = \frac{1}{16}$

$\left(2^5\right)^{x-2} = \frac{1}{2^4}$

$2^{5x-10} = 2^{-4}$

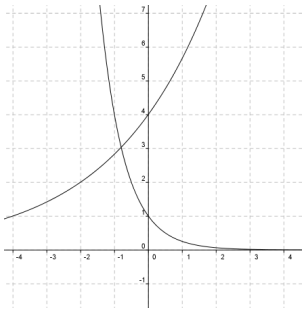
$5x - 10 = -4$

$5x = 6$

$x = 1\frac{1}{5}$

Opgave 41:

a.



b. $\left(\sqrt{2}\right)^{x+4} = \left(\frac{1}{4}\right)^x$

$\left(2^{\frac{1}{2}}\right)^{x+4} = \left(\frac{1}{2^2}\right)^x$

$2^{\frac{1}{2}x+2} = \left(2^{-2}\right)^x$

$2^{\frac{1}{2}x+2} = 2^{-2x}$

$\frac{1}{2}x + 2 = -2x$

$2\frac{1}{2}x = -2$

$x = -\frac{4}{5}$

dus $x \geq -\frac{4}{5}$

c. $\left(\frac{1}{4}\right)^x = \sqrt{2}$

$2^{-2x} = 2^{\frac{1}{2}}$

$-2x = \frac{1}{2}$

$x = -\frac{1}{4}$

dus $x \leq -\frac{1}{4}$

7.4 Logaritmen

Opgave 42:

a. $:3$

b. $\sqrt[3]{\ }$

Opgave 43:

a. $2^x = 3$

$x = {}^2\log 3$

b. $\left(\frac{1}{2}\right)^x = 7$

$x = {}^{\frac{1}{2}}\log 7$

Opgave 44:

a. ${}^2\log 5 = \dfrac{\log 5}{\log 2} = 2{,}32$

b. ${}^4\log 0{,}6 = \dfrac{\log 0{,}6}{\log 4} = -0{,}37$

Opgave 45:

a. $2^{x-1} = 15$

$x - 1 = {}^2\log 15$

$x = 1 + {}^2\log 15$

b. $1 + 2^x = 15$

$2^x = 14$

$x = {}^2\log 14$

c. $4 + 3^{x+1} = 25$

$3^{x+1} = 21$

$x + 1 = {}^3\log 21$

$x = -1 + {}^3\log 21$

d. $14 - 2^{x+3} = 2$

$-2^{x+3} = -12$

$2^{x+3} = 12$

$x + 3 = {}^2\log 12$

$x = -3 + {}^2\log 12$

e. $7 + 4^{2x} = 12$

$4^{2x} = 5$

$2x = {}^4\log 5$

$x = \frac{1}{2} \cdot {}^4\log 5$

f. $3 \cdot 5^{x+1} = 60$

$5^{2x+1} = 20$

$2x + 1 = {}^5\log 20$

$2x = -1 + {}^5\log 20$

$x = -\frac{1}{2} - \frac{1}{2} \cdot {}^{5}\log 20$

Opgave 46:

$2^x = 32$ heeft als oplossing $x = {}^{2}\log 32$

$2^x = 32$ heeft als oplossing $x = 5$ want $2^5 = 32$

dus ${}^{2}\log 32 = 5$

Opgave 47:

a. ${}^{2}\log 4 = {}^{2}\log 2^2 = 2$

b. ${}^{2}\log 2 = {}^{2}\log 2^1 = 1$

c. ${}^{2}\log \frac{1}{2} = {}^{2}\log 2^{-1} = -1$

d. ${}^{2}\log \sqrt{2} = {}^{2}\log 2^{\frac{1}{2}} = \frac{1}{2}$

e. ${}^{2}\log \frac{1}{4} = {}^{2}\log \frac{1}{2^2} = {}^{2}\log 2^{-2} = -2$

f. ${}^{2}\log 1 = {}^{2}\log 2^0 = 0$

g. ${}^{2}\log 4\sqrt{2} = {}^{2}\log(2^2 \cdot 2^{\frac{1}{2}}) = {}^{2}\log 2^{2\frac{1}{2}} = 2\frac{1}{2}$

h. ${}^{2}\log \frac{1}{8}\sqrt{2} = {}^{2}\log(\frac{1}{2^3} \cdot 2^{\frac{1}{2}}) = {}^{2}\log 2^{-2\frac{1}{2}} = -2\frac{1}{2}$

Opgave 48:

a. ${}^{3}\log 27 = {}^{3}\log 3^3 = 3$

b. ${}^{7}\log 49 = {}^{7}\log 7^2 = 2$

c. ${}^{3}\log \frac{1}{81} = {}^{3}\log \frac{1}{3^4} = {}^{3}\log 3^{-4} = -4$

d. ${}^{10}\log 1000 = {}^{10}\log 10^3 = 3$

e. ${}^{10}\log 0{,}01 = {}^{10}\log 10^{-2} = -2$

f. ${}^{10}\log 0{,}1\sqrt{10} = {}^{10}\log(10^{-1} \cdot 10^{\frac{1}{2}}) = {}^{10}\log 10^{-\frac{1}{2}} = -\frac{1}{2}$

g. ${}^{7}\log 1 = {}^{7}\log 7^0 = 0$

h. ${}^{23}\log 23 = {}^{23}\log 23^1 = 1$

Opgave 49:

a. ${}^{5}\log 0{,}2 = {}^{5}\log \frac{1}{5} = {}^{5}\log 5^{-1} = -1$

b. ${}^{3}\log 3\sqrt{3} = {}^{3}\log(3^1 \cdot 3^{\frac{1}{2}}) = {}^{3}\log 3^{1\frac{1}{2}} = 1\frac{1}{2}$

c. ${}^{\frac{1}{2}}\log 8 = {}^{\frac{1}{2}}\log 2^3 = {}^{\frac{1}{2}}\log(2^{-1})^{-3} = {}^{\frac{1}{2}}\log(\frac{1}{2})^{-3} = -3$

d. ${}^{\frac{1}{4}}\log \frac{1}{16} = {}^{\frac{1}{4}}\log(\frac{1}{4})^2 = 2$

e. ${}^{0{,}25}\log 4 = {}^{\frac{1}{4}}\log(4^{-1})^{-1} = {}^{\frac{1}{4}}\log(\frac{1}{4})^{-1} = -1$

f. ${}^{4}\log 0{,}25 = {}^{4}\log \frac{1}{4} = {}^{4}\log 4^{-1} = -1$

g. ${}^{\frac{1}{7}}\log 7 = {}^{\frac{1}{7}}\log(7^{-1})^{-1} = {}^{\frac{1}{7}}\log(\frac{1}{7})^{-1} = -1$

h. ${}^{\frac{1}{7}}\log 1 = {}^{\frac{1}{7}}\log(\frac{1}{7})^0 = 0$

Opgave 50:

a. $^2\log x = 8$

$x = 2^8 = 256$

b. $^3\log x = 1$

$x = 3^1 = 3$

c. $^x\log 3 = 1$

$x^1 = 3$

$x = 3$

d. $^2\log(x+3) = -1$

$x + 3 = 2^{-1}$

$x + 3 = \frac{1}{2}$

$x = -2\frac{1}{2}$

e. $^{\frac{1}{2}}\log(x - \frac{1}{2}) = -1$

$x - \frac{1}{2} = (\frac{1}{2})^{-1}$

$x - \frac{1}{2} = (2^{-1})^{-1}$

$x - \frac{1}{2} = 2$

$x = 2\frac{1}{2}$

f. $^3\log(x^2 + 1) = 2$

$x^2 + 1 = 3^2$

$x^2 + 1 = 9$

$x^2 = 8$

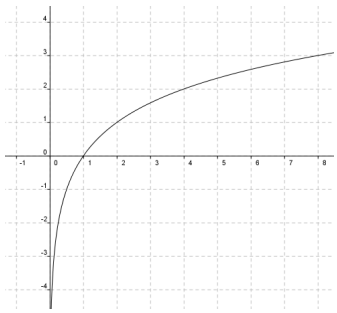
$x = \sqrt{8} \quad \vee \quad x = -\sqrt{8}$

Opgave 51:

a.

x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
$f(x)$	-3	-2	-1	0	1	2	3

b.



c. $y = ^2\log x$

$x = 2^y$

$2^y > 0$ voor iedere y dus $x > 0$

d. $B_f = R$

Opgave 52:

a. $y={}^{3}\log x \xrightarrow{T(-2,0)} y={}^{3}\log(x+2)$

b. $y={}^{2}\log x \xrightarrow{Vx-as,5} y=5\cdot {}^{2}\log x \xrightarrow{T(1,0)} y=5\cdot {}^{2}\log(x-1)$

c. $y={}^{\frac{1}{2}}\log x \xrightarrow{Vx-as,4} y=4\cdot {}^{\frac{1}{2}}\log x \xrightarrow{T(0,3)} y=3+4\cdot {}^{\frac{1}{2}}\log x$

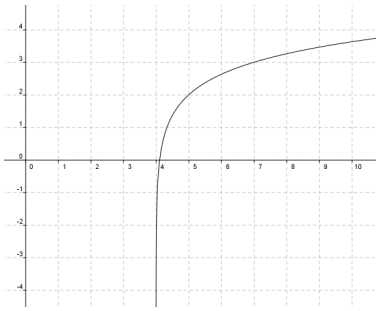
d. $y={}^{\frac{1}{3}}\log x \xrightarrow{Vx-as,-1} y=-1\cdot {}^{\frac{1}{2}}\log x \xrightarrow{T(-1,-2)} y=-2-1\cdot {}^{\frac{1}{3}}\log(x+1)$

e. $y={}^{3}\log x \xrightarrow{Vy-as,\frac{1}{2}} y={}^{3}\log 2x \xrightarrow{T(0,5)} y=5+{}^{3}\log 2x$

f. $y={}^{\frac{1}{4}}\log x \xrightarrow{Vy-as,2} y={}^{\frac{1}{4}}\log \frac{1}{2}x \xrightarrow{Vx-as,3} y=3\cdot {}^{\frac{1}{4}}\log \frac{1}{2}x$

Opgave 53:

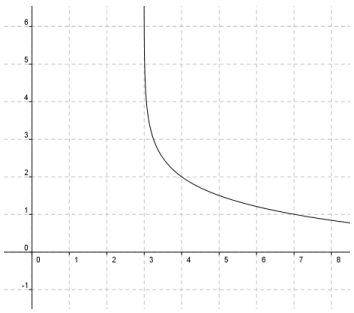
a. $T(4,2)$

b. 

$D_f =< 4,\rightarrow >$

Opgave 54:

a. $T(3,2)$

b. 

$D_f =< 3,\rightarrow >$

Opgave 55:

a. $y={}^{3}\log x \xrightarrow{Vx-as,2} y=2\cdot {}^{3}\log x \xrightarrow{T(0,-4)} y=-4+2\cdot {}^{3}\log x$

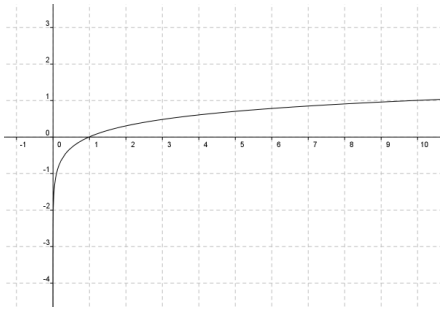
b. $y={}^{3}\log x \xrightarrow{Sx-as} y={}^{3}\log(-x) \xrightarrow{T(5,0)} y={}^{3}\log(-(x-5))={}^{3}\log(-x+5)$

c. $y={}^{3}\log x \xrightarrow{T(-3,-2)} y=2+{}^{3}\log(x+3) \xrightarrow{Vx-as,\frac{1}{2}} =1+\frac{1}{2}\cdot {}^{3}\log(x+3)$

Opgave 56:

a.

x	$\frac{1}{2}$	1	2	5	10
$f(x)$	$-0{,}3$	0	0,3	0,7	1

b. 

c. $X\min = 0 \quad X\max = 10 \quad Y\min = -1 \quad Y\max = 1$

Opgave 57:

a. $21 = 1 + k \cdot \log 100$

$20 = k \cdot 2$

$k = 10$

b. $Din = 1 + 10 \cdot \log 400 = 27$

c. $24 = 1 + 10 \cdot \log ASA$

$23 = 10 \cdot \log ASA$

$\log ASA = 2{,}3$

$ASA = 10^{2{,}3} = 200$

7.5 Diagnostische toets

Opgave 1:

a. $6a^3 \cdot 8(a^2)^2 = 48a^3 \cdot a^4 = 48a^7$

b. $(6a)^3 \cdot (8a^2)^2 = 216a^3 \cdot 64a^4 = 13824a^7$

c. $\dfrac{(6a^2)^3}{(2a)^4} = \dfrac{216a^6}{16a^2} = 13\frac{1}{2}a^2$

d. $(2a^2)^4 - (3a^3)^2 = 16a^8 - 9a^6$

e. $(ab^2)^4 \cdot a^2b = a^4b^8 \cdot a^2b = a^6b^9$

f. $\left(\dfrac{6a^2}{2a}\right)^4 = (3a)^4 = 81a^4$

Opgave 2:

a. $a^{-3} \cdot a^2 = a^{-1}$

b. $(a^{-3})^2 = a^{-6}$

c. $\dfrac{a^{-3}}{a^2} = a^{-5}$

Opgave 3:

a. $a^{-2} = \dfrac{1}{a^2}$

b. $10ab^{-2} = \dfrac{10a}{b^2}$

c. $(4a)^{-2} \cdot 3b^{-4} = \dfrac{1}{(4a)^2} \cdot \dfrac{3}{b^4} = \dfrac{3}{16a^2b^4}$

Opgave 4:

a. $3\frac{1}{2}a^{\frac{2}{7}} = 3\frac{1}{2} \cdot \sqrt[7]{a^2}$

b. $2a^{-3}b^{\frac{1}{3}} = \dfrac{2 \cdot \sqrt[3]{b}}{a^3}$

c. $4a^{\frac{1}{4}}b^{-\frac{2}{3}} = \dfrac{4 \cdot \sqrt[4]{a}}{b^{\frac{2}{3}}} = \dfrac{4 \cdot \sqrt[4]{a}}{\sqrt[3]{b^2}}$

Opgave 5:

a. $\dfrac{1}{x^3} = x^{-3}$

b. $\dfrac{1}{x^2 \cdot \sqrt{x}} = \dfrac{1}{x^2 \cdot x^{\frac{1}{2}}} = \dfrac{1}{x^{2\frac{1}{2}}} = x^{-2\frac{1}{2}}$

c. $\sqrt[3]{x^2} = x^{\frac{2}{3}}$

d. $x^3 \cdot \sqrt[5]{x^3} = x^3 \cdot x^{\frac{3}{5}} = x^{3\frac{3}{5}}$

e. $\dfrac{x^4}{\sqrt[3]{x}} = \dfrac{x^4}{x^{\frac{1}{3}}} = x^{3\frac{2}{3}}$

f. $\sqrt[3]{\frac{1}{x^3}} = \sqrt[3]{x^{-3}} = x^{-\frac{3}{3}} = x^{-1}$

Opgave 6:

a. $16\sqrt{2} = 2^4 \cdot 2^{\frac{1}{2}} = 2^{4\frac{1}{2}}$

$\sqrt[3]{32} = \sqrt[3]{2^5} = 2^{\frac{5}{3}}$

$\sqrt[5]{\frac{1}{8}} = \sqrt[5]{\frac{1}{2^3}} = \sqrt[5]{2^{-3}} = 2^{-\frac{3}{5}}$

b. $2^{x-4} = 2^{-4} \cdot 2^x = \frac{1}{2^4} \cdot 2^x = \frac{1}{16} \cdot 2^x$

$2^{x+\frac{1}{2}} = 2^{\frac{1}{2}} \cdot 2^x = \sqrt{2} \cdot 2^x$

c. $2{,}16^{a-1} = 2{,}16^{-1} \cdot 2{,}16^a = 0{,}46 \cdot 2^a$

$1{,}27^{3a+0{,}6} = 1{,}27^{0{,}6} \cdot 1{,}27^{3a} = 1{,}15 \cdot (1{,}27^3)^a = 1{,}15 \cdot 2{,}05^a$

Opgave 7:

a. $5x^{1,2} + 6 = 20$

$5x^{1,2} = 14$

$x^{1,2} = 2{,}8$

$x = \sqrt[1,2]{2{,}8} = 2{,}358$

b. $6 \cdot \sqrt[3]{x^2} + 3 = 8$

$6 \cdot \sqrt[3]{x^2} = 5$

$\sqrt[3]{x^2} = \frac{5}{6}$

$x^2 = \left(\frac{5}{6}\right)^3$

$x^2 = \frac{125}{216}$

$x = \sqrt{\frac{125}{216}} = 0{,}761$

c. $8x\sqrt{x} + 5 = 21$

$8x\sqrt{x} = 16$

$x\sqrt{x} = 2$

$x^3 = 4$

$x = \sqrt[3]{4} = 1{,}587$

Opgave 8:

a. $K = a \cdot p^{1,3}$

$150 = a \cdot 17^{1,3}$

$150 = 39{,}8a$

$a = 3{,}77$

b. $N = a \cdot \frac{1}{t^{0,83}}$

$33 = a \cdot \frac{1}{11^{0,83}}$

$33 = 0{,}14a$

$a = 241$

Opgave 9:

a. $B = <\leftarrow, 800>$

H.A.: $N = 800$

b. $B = <3, \rightarrow>$

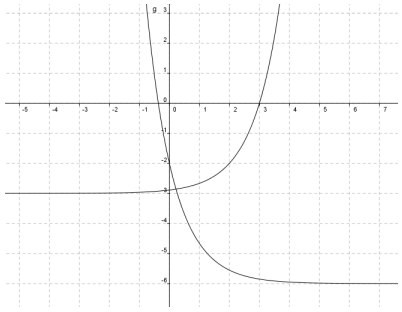
H.A.: $N = 800$

Opgave 10:

a. $y = 3^x \xrightarrow{T(2,-3)} = \quad y = 3^{x-2} - 3$

$y = (\frac{1}{3})^x \xrightarrow{Vx-as,4} y = 4 \cdot (\frac{1}{3})^x \xrightarrow{T(0,-6)} y = 4 \cdot (\frac{1}{3})^x - 6$

b.



$B_f = <-3, \rightarrow>$

$B_g = <-6, \rightarrow>$

c. $y_1 = 3^{x-2} - 3$ en $y_2 = 4 \cdot (\frac{1}{3})^x - 6$ intersect geeft $x = 0{,}22$

dus $x \geq 0{,}22$

d. $p \leq -3$

Opgave 11:

a. $5^{x-1} = 125$

$5^{x-1} = 5^3$

$x - 1 = 3$

$x = 4$

b. $3^{2x-5} = \frac{1}{27}$

$3^{2x-5} = \frac{1}{3^3}$

$3^{2x-5} = 3^{-3}$

$2x - 5 = -3$

$2x = 2$

$x = 1$

c. $2 \cdot 4^{2x-1} - 3 = 61$

$2 \cdot 4^{2x-1} = 64$

$4^{2x-1} = 32$

$(2^2)^{2x-1} = 2^5$

$2^{4x-2} = 2^5$

$4x - 2 = 5$

$4x = 7$

$x = 1\frac{3}{4}$

Opgave 12:

a. $7^{x-3} = 20$

$x - 3 = {}^{7}\log 20$

$x = 3 + {}^{7}\log 20$

b. $6 \cdot 2^{x} + 5 = 23$

$6 \cdot 2^{x} = 18$

$2^{x} = 3$

$x = {}^{2}\log 3$

c. $10 \cdot (\frac{1}{2})^{2x-1} = 600$

$(\frac{1}{2})^{2z-1} = 60$

$2x - 1 = {}^{\frac{1}{2}}\log 60$

$2x = 1 + {}^{\frac{1}{2}}\log 60$

$x = \frac{1}{2} + \frac{1}{2} \cdot {}^{\frac{1}{2}}\log 60$

Opgave 13:

a. ${}^{2}\log 256 = {}^{2}\log 2^{8} = 8$

b. ${}^{3}\log 3\sqrt{3} = {}^{3}\log(3^{1} \cdot 3^{\frac{1}{2}}) = {}^{3}\log 3^{1\frac{1}{2}} = 1\frac{1}{2}$

c. ${}^{5}\log \frac{1}{25} = {}^{5}\log \frac{1}{5^{2}} = {}^{5}\log 5^{-2} = -2$

Opgave 14:

a. ${}^{2}\log x = -3$

$x = 2^{-3} = \frac{1}{2^{3}} = \frac{1}{8}$

b. ${}^{3}\log(x-4) = 2$

$x - 4 = 3^{2} = 9$

$x = 13$

c. ${}^{4}\log(x^{2} - 5) = 1$

$x^{2} - 5 = 4^{1}$

$x^{2} = 9$

$x = 3 \quad \vee \quad x = -3$

Opgave 15:

a. $y = {}^{2}\log x \xrightarrow{T(-5,0)} y = {}^{2}\log(x+5)$

$y = {}^{\frac{1}{2}}\log x \xrightarrow{Vy-as,\frac{1}{2}} y = {}^{\frac{1}{2}}\log(2x) \xrightarrow{T(0,-4)} y = {}^{\frac{1}{2}}\log(2x) - 4$

b. $D_f = <-5, \rightarrow>$

$D_g = <0, \rightarrow>$